ABSTRACT

Voltage stability has always been a very important issue in the planning and operation of power systems, but today it is much more serious. Power systems are in a transition period. The conventional centralized configuration of the power system has been transforming into a decentralized and deregulated form. The electrification of different sectors is accelerating the increase in demand for electrical energy. Increasing energy demand, differentiating consumption profiles, and diversifying consumers have made power system operations more complex. All these transformations take place in the shadow of rising economic and environmental concerns. Therefore, a general approach has been adopted to derive the greatest possible benefit from existing systems, rather than constructing new facilities. As a result, it is aimed to operate the systems close to limit values as much as possible.

The voltage stability of systems operating close to the limit values needs to be examined more precisely. There are many voltage stability indices in the literature. However, most of them are based on various ignorance or assumptions. In this study, a new index is proposed, which is based on line loading and considering multiple line parameters. It has been observed that the suggested index gives consistent results in all varying conditions and different loading types.

Cite this article as: Doğanşahin K, Çikan M. A new line stability index for voltage stability analysis based on line loading. Clean Energy Technol 2023;1(1):23–30.

INTRODUCTION

Voltage stability is an essential issue in the planning and operation of power systems. Voltage stability denotes “the ability of the power system to maintain steady voltages all buses in the system after being subjected to a disturbance” [1]. Loss of supply-demand balance, switching events, generators exceeding reactive limits or loss of function of on-load tap changer transformers are some of the disturbances that cause voltage instability in power systems.

A new voltage stability index
An approach by using quadratic equation of the two-bus network model
Voltage stability assessment based on line loadability

Voltage stability analyzes are performed on planning and operation of the power systems in order to minimize the risk of voltage instability. Two main approaches are
generally adopted for voltage stability analysis. The first, and also the most former, is on the examination of PV and QV curves. This approach is a computationally demanding option as it requires sequential power flow analyses. The second approach is to make an estimation by using voltage stability indices (VSIs). The analyzes by using indices are faster and require less computational effort. In today's power systems, even at the distribution level, it has become possible to take instantaneous measurements from the system with phasor measurement units (PMUs), collect the measurements in a center, process the collected data, and send the control signals to the system. It is important that all these operations need to be implemented promptly, with the data, which are less, and accurate, as much as possible [2]. With these aspects, voltage stability indices are widely used especially in the operation of power systems.

In the literature, there are quite up-to-date and comprehensive survey studies on voltage stability indices. In the review study performed by Salama et al., the application areas of VSIs and their use in various problems have been examined comprehensively [2]. In the paper, a summary table is presented with information about the equations, critical values, assumptions on which they are based, and pros and cons of the 48 VSIs. In the study by Modaressi et al., the assumptions, equations and critical values of the indices based on the derivation of the VSIs have been examined in detail [3]. In another study, the performances of the VSIs proposed in the last three decades have been tested on two different test system with different case studies and the results have been compared [4]. When the above-mentioned studies are examined, it is seen that most of the studies in the literature aim to obtain VSIs in the form of simple expressions that can give approximate estimates about the state of the system. There are 48 different VSIs in the table compiled from the studies examined within the scope of Reference [2]. In half of these 48 VSIs power components have not been taken into account. On the other hand, line charging current, which is an important support for voltage stability, has not been ignored in the derivation of only 8 VSIs.

The transition process in power systems, increasing electrical energy supply, participation of new types of consumers into the system, and diversifying consumption profiles have made the operation of power systems more complicated. On the other hand, obtaining the highest possible benefit from the systems is essential due to the increasing economic and environmental sensitivities in the market. Therefore, power systems are operated near critical points more than ever before. This has made power systems more susceptible to disturbances and has forced operators to be more precise in planning and calculations regarding the operation of the system. The effect of this issue is also felt in the literature that the number of studies performed in recent years to obtain VSIs based on less negligence and assumptions has been increasing. Mokred et al. has proposed a VSI in which the components of the power delivered and the impedance of the system are included [5]. In another recent study, in addition to the components considered in [5], a VSI has been proposed that takes into account the phase difference between the line sending and receiving voltages [6]. In a different study, in order to increase the sensitivity of the FVSI index, which is widely accepted in the literature, it has been modified by including the active power component that has not been included in the original expression of the index [7].

In this study, a new voltage stability index based on system loadability has been proposed. In the mathematical expression of the proposed voltage stability index, the components of the delivered power and the system impedance take place together with the line charging currents. In this aspect, the proposed index is differentiating from others with the number of parameters has been considered in its derivation. In addition, the parameters of power lines may vary by being affected by environmental conditions [8]. The proposed index is quite functional in terms of observing the effect of the variation of the line parameters on voltage stability.

In the second section of the study, the steps followed in the derivation of the proposed voltage stability index have been given. In order to test the performance of the proposed VSI, various case studies have been performed on the 118 bus IEEE test system. In the third part the result obtained from analyzes have been shared comparatively with the results for different VSIs and the finding have been discussed. In the last part of the study, a brief information about the study and remarkable findings are given.

PROPOSED VOLTAGE STABILITY INDEX

In the derivation of the proposed voltage stability index, the single line diagram given in Figure 1 is taken as the basis. Where $S_r$ is the receiving end apparent power, $P_r$ and $Q_r$ are the active and reactive power components of this value. $V_s$ and $V_r$ represent the amplitudes of the sending and receiving end voltages, respectively. $\delta_s$ and $\delta_r$ are for the phase angles of these voltages. $\delta$ indicates the angular difference between the voltages ($\delta = \delta_s - \delta_r$). $R$ and $X$ are for the resistance and reactance values of the line, $Z$ and $\theta$ express the amplitude and the angle of the line impedance.

Figure 1. Considered model of a transmission line.
The proposed voltage stability index has been obtained using the quadratic equation given by Equation 1. The expressions for the ABCD variables in the equation are given in Equation 2–5.

\[
\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}
\]  
\(A = 1 + (Y \times \frac{Z}{2})\)  
\(B = Z\)  
\(C = Y \times \left[ 1 + \left( Y \times \frac{Z}{4} \right) \right]\)  
\(D = A\)

The current injected into the system from the sending end can be expressed with the voltage and current at the receiving end by Equation 6.

\(I_S = C \times V_R + D \times I_R\)

The expressions for the currents in terms of apparent power and voltage variables are given by Equation 7–8.

\[
\frac{|S_s|^2}{|V_s|^2} \leq \frac{(|V_R| - \frac{\phi_s}{2}) \times (|V_R| - \frac{\phi_s}{2}) + (|V_R| - \frac{\phi_s}{2}) \times |V_R| - \frac{\phi_s}{2}}{|V_R| - \frac{\phi_s}{2}}
\]

\[
|V_s| \times |V_R| \times (|V_R| - \frac{\phi_s}{2}) + |V_s| \times |V_R| \times (|V_R| - \frac{\phi_s}{2}) = 0
\]

where \(\gamma\) and \(\alpha\) are the angles of the C and D variables, which is given in Equation 4 and Equation 5. The angular phase difference between voltage and current, also known as the power angle, is denoted by \(\phi\). Equation 9 is obtained when the active parts of the expressions on both sides of the Equation 8 are equalized.

\[
(4|V_s|^2|C||D|\cos(\alpha - \phi_s - \delta_s) \cos(\gamma - \delta_s)) \leq 1
\]

To simplify the equation, the angles of the parameters C and D can be assumed as 90 and 0 degrees, respectively. Thus the equation for the proposed voltage stability index has been obtained as given in Equation 13.

\[
\frac{4|V_s|^2|C||D|\cos(\phi_s + \delta_s) \cdot \sin(\delta_s)}{|S_s|^2 \cdot \cos^2(\phi_s + \delta_s)} \leq 1
\]

where C and D are the line constants. The sending end voltage, on the other hand, can vary in a small range. The index value being less than or equal to one means that there is an acceptable set of solutions for the end-of-line voltage. Otherwise, it indicates voltage instability in the system. On the other hand, apparent powers of receiving and sending ends constitute a proportional relationship in the mathematical expression, as \((S_R/S_s)\). This ratio is always less than one in real systems. Besides, since the sending end apparent power is located in the denominator with its second power, it has a dominant effect on the index value. Obviously, as the sending end apparent power increases, the index value will approach zero. In this case, we can say that the value of the proposed index approaches zero at the maximum load point.

The line admittance is included as a coefficient in the variable C in the numerator of the proposed index. In cases where the admittance value is neglected and taken as zero, the suggested index will be directly equal to zero. In these conditions, the index loss its function. Most of the VSIs in the literature have several similar drawbacks. However, with the line admittance value included in the index, an expression with high voltage sensitivity has been obtained. In this study, different cases have been studied to test the performance of the proposed index. The performance of the proposed index have been compared with that of different VSIs in the next section.

**CASE STUDIES**

In order to test the performance of the proposed voltage stability index IEEE 118-bus test system [9] have been used. The obtained results have been compared with the results of 5 different indices: \(L_{nu}\) (Line Stability Index) [10], \(FVSI\) (Fast Voltage Stability Index) [11], \(LQP\) (Line Quality Proximity Index) [12], \(LCPI\) (Line Collapse Proximity Index) [13] and \(LVSI\) (Line Voltage Stability Indicator) [14]. Calculated values for the chosen VSIs over the results from the power flow analysis for the base case of the test system have been given in Table 1.

In the case analysis based on the base values of the test system, the lowest value for the proposed index has been obtained for the line between the 26 and 30 buses. The lowest value obtained for LVSI, derived by using ABCD
parameters of the transmission line as similar to the proposed index, belongs to the line determined by the proposed index. For the other voltage stability indices, Lmn, FVSI, LQP and LCPI, the line between the 38 - 65 buses, has been determined as the one that is closest to instability. The difference between the proposed index and these indices may be explained by the fact that the admittance of the line is neglected in the calculations. Only in the LVSI, the admittance of the line is taken into consideration. Therefore, the consistency between the proposed index and the LVSI has been examined more closely in the following case studies.

The value of the proposed index for the line between 69 - 70 buses is 0. This is because the admittance value of this line is 0. The reason for having some results with negative sign is because of opposite signs of the trigonometric expressions in the nominator part of the equation.

In Table 2, the results obtained from the voltage stability indices for the heavy active loading case have been shared. The load of the selected bus is increased until the power flow algorithm cannot converge. The values of indices for the highest loading factor and the address of the critical lines have been shared in Table 3. The proposed index has yielded very successful results for heavy active loading case. While all of the other indices have been out of the
limit values at the highest active loading factor, the proposed index has achieved to remain below the value of 1. The critical line shared in the table for the LVSI index is the line with the smallest value in the defined range for the LVSI index ($2 > \text{LVSI} > 1$). However, the LVSI index have the values out of this range. When the results are examined, it is seen that the critical lines indicated by the proposed index are the lines loaded more than the lines indicated by the other indices.

In the following case, the MVA loads of all buses in IEEE 118 bus test system have been increased simultaneously until the power flow algorithm cannot converge. The system has been loaded till 1.81 times base loading. According to the result obtained from the proposed index, the line between the 4 and 5 buses is the most critical line in the system at maximum MVA loading. In Table 3, the results of the indices for the line between the 4 and 5 buses in the case of MVA loading are given.

In order to have a better understanding on the behavior of the proposed VSI, the response of the index against the variation of active loading of receiving end bus has been examined. The LVSI index has been also subjected to the
same examination. At first bus 117 has been selected for the examination. It is at the end of a radial line and only supplied from Bus 12. The active power of the bus at the receiving end has been increased step by step till the 13 times of the base loading and the results have been plotted for each step in Figure 2.

In the Figure 2, the proposed index has been approached from the negative side towards zero. At the initial step, LVSI have given a value around its upper limit of two. By each step it has approached its lower limit value of one. It has reached the value of one at eight times the base loading and then continued its decline.

The line between the buses 12 - 117 have been an example of a radial system. In order to make the same examination for a line in network structure, bus - 50 has been selected. Bus - 50 is located in a ring within the network structure of 118 bus test system and it is connected to buses of 49 and 57. The active power of the Bus - 50 has been increased step by step until the power flow analysis could not converge. The obtained results have been presented in Figures 3 and 4.

As can be seen, the proposed index moved away from 0 for a while, and then it tended to zero again. Likewise, the LVSI also progressed towards 2 for a while and then turned back to 1. The reason for this behavior is the change of power flow direction. In the base loading, the power flows from Bus - 50 to Bus - 57 until the moment of loading factor reached 10, the power flow is against that direction for greater loading factors. In the face of this change, the proposed index gave very consistent results.

In the Figure 2, the proposed index has been approached from the negative side towards zero. At the initial step, LVSI have given a value around its upper limit of two. By each step it has approached its lower limit value of one. It has reached the value of one at eight times the base loading and then continued its decline.

The line between the buses 12 - 117 have been an example of a radial system. In order to make the same examination for a line in network structure, bus - 50 has been selected. Bus - 50 is located in a ring within the network structure of 118 bus test system and it is connected to buses of 49 and 57. The active power of the Bus - 50 has been increased step by step until the power flow analysis could not converge. The obtained results have been presented in Figures 3 and 4.

As can be seen, the proposed index moved away from 0 for a while, and then it tended to zero again. Likewise, the LVSI also progressed towards 2 for a while and then turned back to 1. The reason for this behavior is the change of power flow direction. In the base loading, the power flows from Bus - 50 to Bus - 57 until the moment of loading factor reached 10, the power flow is against that direction for greater loading factors. In the face of this change, the proposed index gave very consistent results.

### Table 3. indices for MVA loading of IEEE 118 bus test system

<table>
<thead>
<tr>
<th>Loading factor</th>
<th>Prop. Index</th>
<th>$L_m$</th>
<th>FVSI</th>
<th>LQP</th>
<th>LCPI</th>
<th>LVSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 1$</td>
<td>$</td>
<td>x</td>
<td>\to 0$</td>
<td>$x &lt; 1$</td>
<td>$x &lt; 1$</td>
<td>$x &lt; 1$</td>
</tr>
<tr>
<td>1.81</td>
<td>0.0013</td>
<td>0.0034</td>
<td>0.0033</td>
<td>0.0039</td>
<td>0.0150</td>
<td>1.0192</td>
</tr>
</tbody>
</table>

**Figure 2.** Variation of Proposed Index and LVSI against active power loading of Bus – 117.

**Figure 3.** LVSI and Proposed index variation for Branch 49 – 50 for loading Bus-50.
CONCLUSION

In the study, a new voltage stability index, which is obtained by using the quadratic equation of the two-bus network model, is proposed. This index has the advantage that it takes into account the line admittance, which has a significant effect on voltage stability. On the other hand, it also considers the active and reactive power balance in the system, as it includes the apparent power of the sending end and receiving end in the calculation. As with all voltage stability indices in the literature, the proposed index has various negative aspects. These are that if the line admittance is zero, the index directly gives the value of zero, and the trigonometric expressions included in the mathematical expression of the index drag the index to undefined at some points. The case studies proved that it would be a more accurate approach to consider more than one voltage stability indices instead of a single index in the voltage stability analysis of systems. Among the voltage stability indices considered within the scope of the study, only LVSI was able to show this compatibility with the proposed voltage stability index. As the future study, the proposed index may be used within algorithms in optimization studies such as DG sizing and placement studies. It may contribute to obtaining more consistent and descriptive results.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES


